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Correlations in a confined magnetized free-electron gas

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Abstract

Equilibrium quantum statistical methods are used to study the pair correlation function for a magnetized free-electron gas in the presence of a hard wall that is parallel to the field. With the help of a path-integral technique and a Green function representation, the modifications in the correlation function caused by the wall are determined both for a non-degenerate and for a completely degenerate gas. In the latter case the asymptotic behaviour of the correlation function for large position differences in the direction parallel to the wall and perpendicular to the field, is found to change from Gaussian in the bulk to algebraic near the wall.

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1. Introduction

As has been known since Bohr *et al* [1], edge effects play an important role in the physics of magnetized charged-particle systems in equilibrium. In particular, the diamagnetic response of these systems in the quantum regime is determined by electric currents flowing near the walls. The profiles of these currents, and of the closely related particle density, for a non-interacting magnetized electron gas near a hard wall parallel to the magnetic field have been investigated in detail [2, 3–6]. Much less is known about the profiles in an interacting magnetized electron gas.

Equally important for a physical understanding of the properties of an equilibrium quantum system are the correlation functions. For positions in the bulk of the system these have been studied extensively, both for a non-interacting magnetized electron gas and for its interacting counterpart. For the non-interacting gas the bulk pair correlation function can be determined analytically both for dilute systems at high temperatures and for dense low-temperature systems, in which quantum degeneracy effects are important [7]. For the interacting electron gas information on the behaviour of the bulk correlation functions is more difficult to obtain. Even for the non-magnetized case these functions have surprising properties.

In fact, it has been demonstrated that the bulk correlation functions of the interacting electron gas possess slowly decaying tails with an algebraic dependence on the position difference [8]. For the magnetized interacting gas similar methods have been employed to prove the existence of analogous algebraic tails, albeit with a different exponent [9].

The correlation functions are expected to change near a hard wall. For a non-magnetized free-particle system these changes are easily determined by using a reflection principle [10]. The problem becomes a lot more complicated when either interactions between the particles or a magnetic field or both are incorporated. In a recent paper [11] the interactions between the charged particles have been taken into account in a model system consisting of two quantum charges immersed in a classical plasma confined by a wall. An algebraic tail in the pair correlation function of the quantum particles near the wall was found. However, the exponent governing the algebraic tail turned out to be different from that of the bulk correlation functions discussed above. The result corroborates earlier findings based on linear response arguments [12]. The influence of a magnetic field on the surface correlation functions in the adopted model system remains to be studied.

As the influence of a wall on the correlations in magnetized quantum systems is not yet fully understood, it appears to be useful to try and investigate the correlation functions for the relatively simple case of a non-interacting magnetized electron gas in the presence of a hard wall. In the following we present some new results for this system. In particular, we will analyse the correlations for the strongly degenerate case of high density and low temperature, where the influence of Fermi statistics is important. Both strong and weak fields will be considered, so that the number of filled Landau levels can vary considerably.

The paper is organized as follows. We start with two sections that serve to prepare the ground. In section 2 we define the relevant correlation functions for a system of independent particles and discuss their relation to the one-particle Green functions. The pair correlation function in the bulk is considered in section 3, where the influence of the magnetic field on the correlations is determined both analytically and numerically. After these preparatory sections we start considering the influence of the wall in section 4. In that section we use the so-called 'path-decomposition expansion', which follows from a path-integral formulation, to determine the lowest-order corrections in the correlation functions at positions in the transition region, where the presence of the wall starts to be felt. An alternative way to determine these corrections is based on an eigenfunction expansion of the Green function, which is the subject of section 5. The asymptotic form of the correlation functions for large position differences is established in section 6, separately for directions parallel with and transverse to the magnetic field. In section 7 the correlation functions for positions close to the wall are studied, again for both directions. In the final section 8 some conclusions will be drawn.

2. Correlations

The equilibrium quantum statistical properties of a system of independent particles are determined by the temperature Green function

$$G_\beta(\boldsymbol{r},\boldsymbol{r}') = \langle \boldsymbol{r}|\mathrm{e}^{-\beta H}|\boldsymbol{r}'\rangle = \sum_n \mathrm{e}^{-\beta E_n}\,\psi_n(\boldsymbol{r})\,\psi_n^*(\boldsymbol{r}'). \tag{1}$$

Here β is the inverse temperature, and $\psi_n(\boldsymbol{r})$ and E_n the eigenfunctions and eigenvalues of the one-particle Hamiltonian H, which is assumed to be independent of the spin of the particles. The reduced single-particle density matrix $\rho_{\beta,\mu}(\boldsymbol{r},\boldsymbol{r}')$ of such a system at inverse temperature

β and chemical potential μ is found by incorporating the effects of quantum degeneracy. For Fermi–Dirac particles one has

$$\rho_{\beta,\mu}(\boldsymbol{r},\boldsymbol{r}') = 2\sum_n \frac{1}{1+\mathrm{e}^{\beta(E_n-\mu)}}\psi_n(\boldsymbol{r})\psi_n^*(\boldsymbol{r}') \tag{2}$$

where the spin degeneracy has been taken into account. The local particle density $\rho_{\beta,\mu}(\boldsymbol{r})$ is the diagonal part of (2).

For a completely degenerate system at zero temperature the reduced single-particle density matrix becomes

$$\rho_\mu(\boldsymbol{r},\boldsymbol{r}') = 2\sum_n \theta(\mu - E_n)\psi_n(\boldsymbol{r})\psi_n^*(\boldsymbol{r}') \equiv G_\mu(\boldsymbol{r},\boldsymbol{r}') \tag{3}$$

with θ the step function. The diagonal part gives the local particle density $\rho_\mu(\boldsymbol{r})$ of the completely degenerate system. The μ-dependent Green function, as defined here, is obtained from the temperature Green function by an inverse Laplace transform [13]

$$G_\mu(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{2\pi\mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}\beta\, \mathrm{e}^{\beta\mu}\frac{2}{\beta}G_\beta(\boldsymbol{r},\boldsymbol{r}') \tag{4}$$

with $c > 0$.

The n-particle reduced density matrix $\rho^{(n)}_{\beta,\mu}(\boldsymbol{r},\boldsymbol{r}')$ follows from its one-particle counterpart by a symmetrized factorization:

$$\rho^{(n)}_{\beta,\mu}(\boldsymbol{r}^n,\boldsymbol{r}'^n) = \sum_{\pi\in\mathcal{S}_m}\epsilon^\pi\prod_{j=1}^n \rho_{\beta,\mu}\left(\boldsymbol{r}_j,\boldsymbol{r}'_{\pi(j)}\right). \tag{5}$$

Here the sum is taken over all permutations of the n position vectors, with ϵ^π the sign of the permutation. The structure of the n-particle reduced density matrix has been analysed quite generally for a system of interacting particles by using a path-integral formalism [14]. The factorization property for a system of independent particles then follows as a special case. The argument is not changed by incorporating an external magnetic field and a hard wall that confines the system.

For the completely degenerate case a relation similar to (5) holds for $\rho^{(n)}_\mu(\boldsymbol{r}^n,\boldsymbol{r}'^n)$. In particular, the diagonal part of the two-point reduced density matrix at zero temperature is

$$\rho^{(2)}_\mu(\boldsymbol{r}\boldsymbol{r}',\boldsymbol{r}\boldsymbol{r}') = G_\mu(\boldsymbol{r},\boldsymbol{r})G_\mu(\boldsymbol{r}',\boldsymbol{r}') - G_\mu(\boldsymbol{r},\boldsymbol{r}')G_\mu(\boldsymbol{r}',\boldsymbol{r}). \tag{6}$$

Often it is convenient to introduce the two-point correlation function

$$g(\boldsymbol{r},\boldsymbol{r}') = \frac{\rho^{(2)}_\mu(\boldsymbol{r}\boldsymbol{r}',\boldsymbol{r}\boldsymbol{r}')}{\rho_\mu(\boldsymbol{r})\rho_\mu(\boldsymbol{r}')} - 1 = -\frac{|G_\mu(\boldsymbol{r},\boldsymbol{r}')|^2}{G_\mu(\boldsymbol{r},\boldsymbol{r})G_\mu(\boldsymbol{r}',\boldsymbol{r}')}. \tag{7}$$

In the following we will study this correlation function, and the influence of a magnetic field and a hard wall on its properties.

3. Correlations in the bulk

We consider a system of charged particles which move in a magnetic field directed along the z-axis. The interaction between the particles is neglected. To describe the magnetic field we adopt the Landau gauge, with vector potential $\boldsymbol{A} = (0, Bx, 0)$. The particles are confined to the half-space $x > 0$ by a plane hard wall at $x = 0$.

For positions far from the wall the temperature Green function $G_\beta(\boldsymbol{r},\boldsymbol{r}')$ reduces to the bulk Green function $G^b_\beta(\boldsymbol{r},\boldsymbol{r}')$. The latter is given by [13]

$$G^b_\beta(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{\sqrt{2\pi\beta}}\frac{B}{4\pi\sinh(\beta B/2)}\exp\left[-\frac{B}{4\tanh(\beta B/2)}(\boldsymbol{r}_\perp-\boldsymbol{r}'_\perp)^2\right] \times\exp\left[\frac{\mathrm{i}B}{2}(x+x')(y-y')\right]\exp\left[-\frac{(z-z')^2}{2\beta}\right]. \tag{8}$$

Units have been chosen such that the charge and the mass of the particles drop out, while $\hbar$ and c have been put to 1 as well. From now on we will often measure distances in terms of the cyclotron radius $1/\sqrt{B}$. To that end we introduce the dimensionless variables $\bar{\xi}=\sqrt{B}(x+x')/2$, $\xi=\sqrt{B}(x-x')$, $\eta=\sqrt{B}(y-y')$ and $\zeta=\sqrt{B}(z-z')$.

The μ-dependent Green function follows by inserting (8) in (4). One finds

$$G^b_\mu(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{2\pi\mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty}\mathrm{d}\beta\,\mathrm{e}^{\beta B\nu}\frac{B}{2^{5/2}\pi^{3/2}\beta^{3/2}}\frac{1-q_0^2}{q_0} \times\exp\left[\mathrm{i}\bar{\xi}\eta-\frac{1+q_0^2}{8q_0}(\xi^2+\eta^2)\right]\exp\left[-\frac{\zeta^2}{2\beta B}\right] \tag{9}$$

with $q_0=\tanh(\beta B/4)$. The chemical potential is measured in terms of the energy difference between adjacent Landau levels, by employing the dimensionless variable $\nu=\mu/B$. Let us use now the generating-function identity [15]

$$\exp\left[-\frac{(1-q_0)^2}{8q_0}\xi^2\right]=\frac{4q_0}{(1+q_0)^2}\sum_{n=0}^{\infty}L_n\left(\frac{\xi^2}{2}\right)\left(\frac{1-q_0}{1+q_0}\right)^{2n} \tag{10}$$

to express the exponential function in terms of Laguerre polynomials. After substitution of $(1-q_0)/(1+q_0)=\mathrm{e}^{-\beta B/2}$ this gives

$$G^b_\mu(\boldsymbol{r},\boldsymbol{r}') = \frac{B^{3/2}}{\sqrt{2}\pi^{3/2}}\exp\left(\mathrm{i}\bar{\xi}\eta-\frac{\xi^2+\eta^2}{4}\right)\sum_{n=0}^{\infty}L_n\left(\frac{\xi^2+\eta^2}{2}\right) \times\frac{1}{2\pi\mathrm{i}}\int_{c'-\mathrm{i}\infty}^{c'+\mathrm{i}\infty}\mathrm{d}s\,\mathrm{e}^{s[\nu-(n+1/2)]}\,s^{-3/2}\exp\left(-\frac{\zeta^2}{2s}\right) \tag{11}$$

where we have set $s=\beta B$ and $c'=cB$. The sum over n can be interpreted as a sum over Landau levels.

The inverse Laplace transform in (11) can be found in [16]:

$$\frac{1}{2\pi\mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty}\mathrm{d}s\,\mathrm{e}^{st}\,s^{-3/2}\,\mathrm{e}^{-a/s}=\frac{1}{\sqrt{\pi a}}\sin\left(2\sqrt{at}\right)\,\theta(t) \tag{12}$$

for $a>0$ and $c>0$. Use of this identity in (11) results in the following expression for the μ-dependent Green function in the bulk:

$$G^b_\mu(\boldsymbol{r},\boldsymbol{r}') = \frac{B^{3/2}}{\pi^2}\exp\left(\mathrm{i}\bar{\xi}\eta-\frac{\xi^2+\eta^2}{4}\right) \times{\sum_n}'L_n\left(\frac{\xi^2+\eta^2}{2}\right)\frac{\sin\left(\sqrt{2[\nu-(n+1/2)]}\,\zeta\right)}{\zeta}. \tag{13}$$

The prime indicates that the summation is only over those values of n for which $\nu-(n+1/2)$ is positive, i.e. over the Landau levels that are at least partially filled. The bulk density follows as the diagonal part of (13):

$$\rho^b_\mu=G^b_\mu(\boldsymbol{r},\boldsymbol{r})=\frac{\sqrt{2}\,B^{3/2}}{\pi^2}{\sum_n}'\sqrt{\nu-(n+1/2)}. \tag{14}$$

The two-particle correlation function is found upon substitution of (13) and (14) in (7).

The expressions (13) and (14) are particularly useful for strong fields when only a few Landau levels are occupied. An alternative expression, which is useful for weak fields only, has been derived in [7]. To study the limit $B \to 0$ in (13), we return to the original variables, since measuring the distances in terms of the cyclotron radius, or the chemical potential in terms of the energy difference between adjacent Landau levels, becomes meaningless for vanishing magnetic fields. The number of terms in the sum becomes large, as the upper limit is inversely proportional to B at fixed μ. Furthermore, the argument of the Laguerre polynomial gets small for fixed $x - x'$ and $y - y'$. Hence, we can use the asymptotic form of the Laguerre polynomials [15]

$$L_n(u) \approx \mathrm{e}^{u/2} J_0\left(\sqrt{2(2n+1)u}\right) \tag{15}$$

which is valid for $u/(n+1/2)^{1/3} \ll 1$. Use of this approximation gives

$$G^b_\mu(\boldsymbol{r},\boldsymbol{r}') \approx \frac{B^{3/2}}{\pi^2} \exp\left[\frac{\mathrm{i}B(x+x')(y-y')}{2}\right] {\sum_n}' J_0\left(\sqrt{2B(n+1/2)}|\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp|\right) \times \frac{\sin\left[\sqrt{2[\mu - B(n+1/2)]}(z-z')\right]}{\sqrt{B}(z-z')} \tag{16}$$

for small magnetic fields. The subscripts $_\perp$ denote the transverse parts of the position vectors, which follow by projection on the xy-plane.

If B approaches zero, the number of Landau levels becomes very large, and their spacing becomes very small. Therefore, it is permitted to replace the summation over Landau levels in (16) with an integral. In the limit of vanishing B we get

$$G^b_\mu(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{\pi^2}\int_0^\mu \mathrm{d}t\; J_0\left(\sqrt{2t}\,|\boldsymbol{r}_\perp - \boldsymbol{r}'_\perp|\right) \frac{\sin\left[\sqrt{2(\mu-t)}(z-z')\right]}{z-z'}. \tag{17}$$

With the help of the identity [17]

$$\int_0^a \mathrm{d}x\, x^{\nu+1} \sin\left(b\sqrt{a^2-x^2}\right) J_\nu(x) = \sqrt{\frac{\pi}{2}} a^{\nu+3/2} b (1+b^2)^{-\nu/2-3/4} J_{\nu+3/2}\left(a\sqrt{1+b^2}\right) \tag{18}$$

for $\nu = 0$, we arrive at

$$G^b_\mu(\boldsymbol{r},\boldsymbol{r}') = \frac{2^{1/4}\mu^{3/4}}{\pi^{3/2}} \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|^{3/2}} J_{3/2}\left(\sqrt{2\mu}|\boldsymbol{r}-\boldsymbol{r}'|\right). \tag{19}$$

Note that the right-hand side is an isotropic function of the position difference, as should be the case for a vanishing magnetic field. We can simplify it further by using the explicit form for the Bessel function

$$J_{3/2}(u) = \sqrt{\frac{2}{\pi}} \frac{\sin u - u\cos u}{u^{3/2}}. \tag{20}$$

The final result is

$$G^b_\mu(\boldsymbol{r},\boldsymbol{r}') = -\frac{1}{\pi^2|\boldsymbol{r}-\boldsymbol{r}'|^2}\left[\sqrt{2\mu}\cos\left(\sqrt{2\mu}|\boldsymbol{r}-\boldsymbol{r}'|\right) - \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}\sin\left(\sqrt{2\mu}|\boldsymbol{r}-\boldsymbol{r}'|\right)\right] \tag{21}$$

which is identical to what one gets by starting from the temperature Green function for the non-magnetized system

$$G^b_\beta(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{(2\pi\beta)^{3/2}} \exp\left[-\frac{(\boldsymbol{r}-\boldsymbol{r}')^2}{2\beta}\right] \tag{22}$$

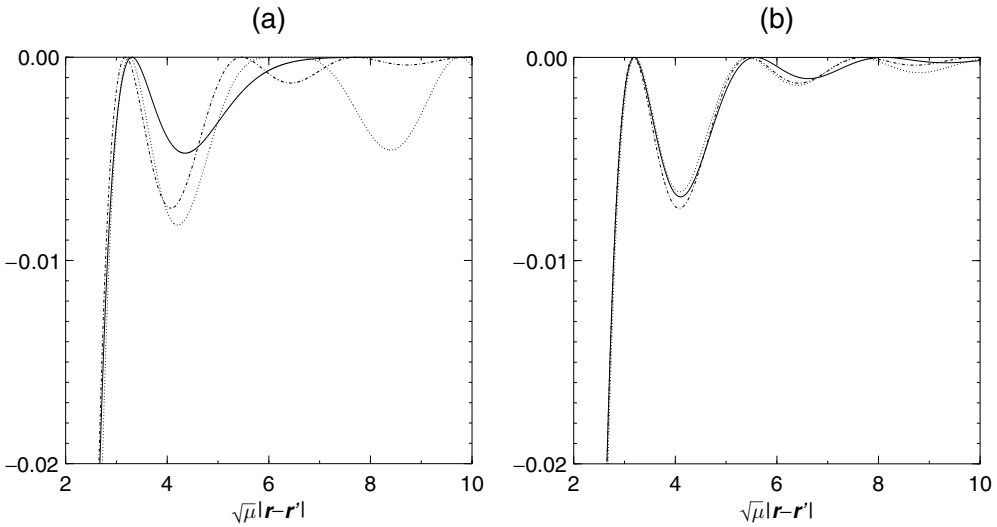

Figure 1. Bulk correlation functions $g(xyz, xy'z)$ (——), $g(xyz, xyz')$ (· · · · · ·) for $B \neq 0$ and (*a*) $\nu = 2$, (*b*) $\nu = 5$, and $g(\boldsymbol{r}, \boldsymbol{r}')$ (— · —) for $B = 0$. All curves start at -1 for $\boldsymbol{r} = \boldsymbol{r}'$.

and applying (4). The bulk density in the field-free case is $\rho_\mu^b = (2\mu)^{3/2}/(3\pi^2)$. The two-particle correlation function in the bulk follows upon inserting (21) in (7).

In figure 1 we have plotted the bulk correlation function for $B = 0$ and for $B \neq 0$ with $\nu = 2$ and $\nu = 5$. For non-vanishing magnetic field we focused on the correlation functions with position differences that are either parallel with, or perpendicular to, the magnetic field. For large fields, or, more precisely, for small ν, the correlation functions for the parallel and the perpendicular directions differ considerably. For somewhat larger ν, however, the correlation functions become fairly similar, both in the nodal structure and in the amplitudes. As it turns out, these similarities are manifest already for $\nu = 5$, where the number of completely filled Landau levels is still rather low.

Comparing (21) with (13), we see that by turning on the magnetic field the range of the correlations in the plane perpendicular to the magnetic field becomes smaller, with a Gaussian instead of an algebraic decay. In contrast, the range of the correlations in the direction parallel to the magnetic field becomes somewhat larger. In fact, although the decay remains algebraic when the field is switched on, the dominant contribution in the tail of the correlation function becomes inversely proportional to the square of the distance, whereas it is inversely proportional to the fourth power of the distance in the field-free case.

4. Path-decomposition expansion

Introducing the wall at $x = 0$ makes the temperature Green function dependent on the distance from the wall, i.e. the coordinate x. In the absence of a magnetic field the influence of the wall on the Green function is easily found from a reflection principle [10]. The temperature Green function becomes

$$G_\beta(\boldsymbol{r}, \boldsymbol{r}') = G_\beta^b(\boldsymbol{r}, \boldsymbol{r}') - G_\beta^b(\boldsymbol{r}, \boldsymbol{r}'') \tag{23}$$

with the bulk Green function (22) and the reflected position $\boldsymbol{r}''$ defined as $(x'', y'', z'') = (-x', y', z')$. Likewise, the μ-dependent Green function gets the form

$$G_\mu(\boldsymbol{r}, \boldsymbol{r}') = G^b_\mu(\boldsymbol{r}, \boldsymbol{r}') - G^b_\mu(\boldsymbol{r}, \boldsymbol{r}'') \tag{24}$$

with the bulk Green function (21).

When a magnetic field is present, the influence of the wall on the properties of the system is more difficult to determine. In [6], we have seen that the temperature Green function can be found from a path-decomposition expansion

$$G_\beta(\boldsymbol{r}, \boldsymbol{r}') = \sum_{n=0}^{\infty} G^{(n)}_\beta(\boldsymbol{r}, \boldsymbol{r}') \tag{25}$$

where $G^{(n)}_\beta(\boldsymbol{r}, \boldsymbol{r}')$ is the contribution from paths that hit the wall n times. This path-decomposition expansion, which was first formulated in [18], is fully equivalent to the multiple-reflection expansion as introduced in [19], and discussed for a confined magnetic system in [4]. We have also seen in [6] that for $\boldsymbol{r}$ and $\boldsymbol{r}'$ at large distances from the wall, the terms with small n in (25) are more important than those with larger n. In particular, the $n = 1$ term will give us the leading-order correction on the bulk quantities. The latter correspond to $n = 0$, so that one has $G^{(0)}_\beta(\boldsymbol{r}, \boldsymbol{r}') = G^b_\beta(\boldsymbol{r}, \boldsymbol{r}')$. If no field is present the expansion terminates after the term with $n = 1$, for all distances from the wall.

In the particular case where $x = x'$, the transverse part of the $n = 1$ term has the form [4, 6]

$$G^{(1)}_{\perp,\beta}(xy, xy') = -\frac{B^2}{16\pi^{3/2}} \int_0^\beta \mathrm{d}\tau \; f^{(1)}_{\beta,\tau}(xy, xy') \exp\left[g^{(1)}_{\beta,\tau}(xy, xy')\right]. \tag{26}$$

The functions $f^{(1)}_{\beta,\tau}(xy, xy')$ and $g^{(1)}_{\beta,\tau}(xy, xy')$ are given by

$$f^{(1)}_{\beta,\tau}(xy, xy') = \frac{1}{2} \frac{(t_1 t_2)^{1/2}}{s_1 s_2 (t_1 + t_2)^{1/2}} \left[\left(\frac{1}{t_1} + \frac{1}{t_2}\right)\bar{\xi} + \mathrm{i}\eta\right] \tag{27}$$

and

$$g^{(1)}_{\beta,\tau}(xy, xy') = -\frac{1}{4}\left[\left(\frac{1}{t_1} + \frac{1}{t_2}\right)\bar{\xi}^2 - 2\mathrm{i}\bar{\xi}\eta + \frac{\eta^2}{t_1 + t_2}\right] \tag{28}$$

with $t_1 = \tanh(\tau B/2)$, $s_1 = \sinh(\tau B/2)$, $t_2 = \tanh((\beta - \tau)B/2)$ and $s_2 = \sinh((\beta - \tau)B/2)$.

4.1. Case $y \neq y'$

If we set $p = q^2/\left(q_0^2 - q^2\right)$, with $q = \tanh[(2\tau - \beta)B/4]$ and $q_0 = \tanh(\beta B/4)$, we can write (26) as

$$\begin{aligned} G^{(1)}_{\perp,\beta}(xy, xy') = &-\frac{B}{2^{7/2}\pi^{3/2}} \frac{1 - q_0^2}{q_0^{3/2}} \exp\left(-\frac{\bar{\xi}^2}{2q_0} + \frac{\mathrm{i}\bar{\xi}\eta}{2} - \frac{1 + q_0^2}{16 q_0}\eta^2\right) \\ &\times \int_0^\infty \frac{\mathrm{d}p}{\sqrt{p(1+p)}} \left[\bar{\xi}\sqrt{1 + \left(1 - q_0^2\right)p} + \frac{\mathrm{i}\eta}{2} \frac{q_0}{\sqrt{1 + \left(1 - q_0^2\right)p}}\right] \\ &\times \exp\left[-\frac{\left(1 - q_0^2\right)p}{2q_0}\bar{\xi}^2 - \frac{1 - q_0^2}{16 q_0} \frac{1 + \left(1 + q_0^2\right)p}{1 + \left(1 - q_0^2\right)p}\eta^2\right]. \end{aligned} \tag{29}$$

When $\bar{\xi}$ is large, only small values of $\left(1 - q_0^2\right)p$ contribute, which implies that we can set $\sqrt{1 + \left(1 - q_0^2\right)p} \approx 1$. Furthermore, we can replace the last exponential function with

$\exp\left[-\left(1-q_0^2\right)\eta^2/(16q_0)\right]$, at least as long as η is finite. If we make those substitutions and use

$$\int_0^\infty \frac{\mathrm{d}p}{\sqrt{p(1+p)}}\,\mathrm{e}^{-ap} = \mathrm{e}^{a/2}K_0(a/2) \tag{30}$$

(where K_0 is the modified Bessel function of the second kind), we get

$$G^{(1)}_{\perp,\beta}(xy,xy') \approx -\frac{B}{2^{7/2}\pi^{3/2}}\left(\bar{\xi}+\frac{\mathrm{i}\eta}{2}q_0\right)\frac{1-q_0^2}{q_0^{3/2}} \times \exp\left(-\frac{1+q_0^2}{4q_0}\bar{\xi}^2+\mathrm{i}\frac{\bar{\xi}\eta}{2}-\frac{\eta^2}{8q_0}\right)K_0\left(\frac{1-q_0^2}{4q_0}\bar{\xi}^2\right). \tag{31}$$

If we are only interested in the non-degenerate case, where in general $(1-q_0^2)\bar{\xi}^2/q_0$ is large, we can use the asymptotic expansion for K_0 [15]. Multiplying the result with the longitudinal Green function, which is the same as in the bulk, we find the first-order correction to the total temperature Green function in the approximate form

$$G^{(1)}_{\beta}(xyz,xy'z) \approx -\frac{B}{2^{7/2}\pi^{3/2}\beta^{1/2}}\frac{\sqrt{1-q_0^2}}{q_0}\exp\left(-\frac{\bar{\xi}^2}{2q_0}+\mathrm{i}\frac{\bar{\xi}\eta}{2}-\frac{\eta^2}{8q_0}\right). \tag{32}$$

However, for the degenerate case we need the full complexity of (31).

In order to obtain results for the degenerate case, we now apply (4). In doing so we choose the contour of integration by setting $\beta = (\mathrm{i}t+1)\bar{\xi}/B$:

$$G^{(1)}_{\mu}(xyz,xy'z) \approx -\frac{B^{3/2}}{16\pi^3\,\bar{\xi}^{1/2}}\mathrm{e}^{\mathrm{i}\bar{\xi}\eta/2}\int_{-\infty}^{\infty}\mathrm{d}t\,\frac{\mathrm{e}^{\nu\bar{\xi}(\mathrm{i}t+1)}}{(\mathrm{i}t+1)^{3/2}}\left(\bar{\xi}+\frac{\mathrm{i}\eta}{2}q_0\right)\frac{1-q_0^2}{q_0^{3/2}} \times \exp\left(-\frac{1+q_0^2}{4q_0}\bar{\xi}^2-\frac{\eta^2}{8q_0}\right)K_0\left(\frac{1-q_0^2}{4q_0}\bar{\xi}^2\right). \tag{33}$$

In the new variables q_0 equals $\tanh[(\mathrm{i}t+1)\bar{\xi}/4]$. From $\bar{\xi}\gg 1$ one finds $q_0\approx 1$, which in turn implies $1-q_0^2 \approx 4\exp[-\bar{\xi}(\mathrm{i}t+1)/2]$. This also means that for finite η we may replace $\bar{\xi}+\mathrm{i}\eta q_0/2$ with $\bar{\xi}$. With the help of the series representation of the modified Bessel function

$$K_0(u) = \sum_{n=0}^{\infty}\left[\sum_{m=1}^{n}\frac{1}{m}-\gamma-\log\left(\frac{u}{2}\right)\right]\frac{1}{2^{2n}(n!)^2}u^{2n} \tag{34}$$

and the integral relation

$$\int_{-\infty}^{\infty}\mathrm{d}t\,\frac{\mathrm{e}^{(\mathrm{i}t+1)x}}{(\mathrm{i}t+1)^{\nu}} = \frac{2\pi\,x^{\nu-1}}{\Gamma(\nu)}\,\theta(x) \qquad (\nu>0) \tag{35}$$

we arrive at the asymptotic expression

$$G^{(1)}_{\mu}(xyz,xy'z) \approx -\frac{B^{3/2}\,\bar{\xi}}{\pi^{5/2}}\,\mathrm{e}^{-\bar{\xi}^2/2}\,\mathrm{e}^{\mathrm{i}\bar{\xi}\eta/2}\,\mathrm{e}^{-\eta^2/8}\sum_n{}'\frac{\bar{\xi}^{4n}}{2^{2n}(n!)^2} \times\sqrt{\nu-(n+1/2)}\left[\frac{1}{4[\nu-(n+1/2)]}+\sum_{m=1}^{n}\frac{1}{m}-\gamma-\ln\left(\frac{\bar{\xi}^2}{2}\right)\right] \tag{36}$$

which is valid for large $\sqrt{B}x$. Again, we recognize the sum over Landau levels. In fact, apart from the phase $\mathrm{e}^{\mathrm{i}\bar{\xi}\eta/2}$ and the factor $\mathrm{e}^{-\eta^2/8}$, the result is identical to what we found in [6] for $\boldsymbol{r}=\boldsymbol{r}'$.

4.2. Case $z \neq z'$

This case is considerably simpler than the $y \neq y'$-case, since the temperature Green function factorizes. The transverse part of the Green functions follows from (31) by taking $\eta = 0$. Using the asymptotic expansion for the Bessel function and introducing the longitudinal part of the Green function, one finds the total temperature Green function for the non-degenerate case with $z \neq z'$ as

$$G_\beta^{(1)}(xyz, xyz') \approx -\frac{B}{2^{7/2}\pi^{3/2}\,\beta^{1/2}} \frac{\sqrt{1-q_0^2}}{q_0} \exp\left(-\frac{\bar{\xi}^2}{2q_0} - \frac{\zeta^2}{2\beta B}\right) \tag{37}$$

which is the analogue of (32).

To calculate the μ-dependent Green function $G_\mu^{(1)}(xyz, xyz')$ for the degenerate case we need to keep the Bessel function in (31). With $s = \beta B$ we get

$$G_\mu^{(1)}(xyz, xyz') \approx -\frac{B^{3/2}\,\bar{\xi}}{8\pi^2} \frac{1}{2\pi\mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}s\, \frac{\mathrm{e}^{\nu s}}{s^{3/2}} \frac{1-q_0^2}{q_0^{3/2}} \exp\left(-\frac{1+q_0^2}{4q_0}\bar{\xi}^2\right)$$
$$\times K_0\left(\frac{1-q_0^2}{4q_0}\bar{\xi}^2\right) \exp\left(-\frac{\zeta^2}{2s}\right). \tag{38}$$

We are free to choose the contour of integration, as long as it is in the right half-plane. By choosing c large (i.e., $c = \bar{\xi}$), we can make the same approximations for q_0 and $1-q_0^2$ as before. In terms of the integration variable s these approximations read $q_0 \approx 1$ and $1 - q_0^2 \approx 4\mathrm{e}^{-s/2}$. Together with (34) this yields

$$G_\mu^{(1)}(xyz, xyz') \approx -\frac{B^{3/2}\,\bar{\xi}}{2\pi^2}\,\mathrm{e}^{-\bar{\xi}^2/2} \sum_{n=0}^{\infty} \frac{\bar{\xi}^{4n}}{2^{2n}(n!)^2} \frac{1}{2\pi\mathrm{i}} \int_{\bar{\xi}-\mathrm{i}\infty}^{\bar{\xi}+\mathrm{i}\infty} \mathrm{d}s\, \mathrm{e}^{[\nu-(n+1/2)]s}$$
$$\times \left\{\frac{1}{2}\frac{1}{s^{1/2}} + \left[\sum_{m=1}^{n}\frac{1}{m} - \gamma - \ln\left(\frac{\bar{\xi}^2}{2}\right)\right]\frac{1}{s^{3/2}}\right\} \exp\left(-\frac{\zeta^2}{2s}\right). \tag{39}$$

The integral in this expression is still an inverse Laplace transform. With the help of (12) and the analogous identity [16]

$$\frac{1}{2\pi\mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} \mathrm{d}s\, \mathrm{e}^{st}\, s^{-1/2}\, \mathrm{e}^{-a/s} = \frac{1}{\sqrt{\pi t}} \cos\left(2\sqrt{at}\right)\theta(t) \tag{40}$$

for $a > 0$ (and $c > 0$) we finally get the asymptotic expression

$$G_\mu^{(1)}(xyz, xyz') \approx -\frac{B^{3/2}\,\bar{\xi}}{\pi^{5/2}}\,\mathrm{e}^{-\bar{\xi}^2/2} {\sum_n}' \frac{\bar{\xi}^{4n}}{2^{2n}(n!)^2} \sqrt{\nu-(n+1/2)}$$
$$\times \left\{\frac{\cos\left(\sqrt{2[\nu-(n+1/2)]}\zeta\right)}{4[\nu-(n+1/2)]}\right.$$
$$\left. + \frac{\sin\left(\sqrt{2[\nu-(n+1/2)]}\zeta\right)}{\sqrt{2[\nu-(n+1/2)]}\zeta} \left[\sum_{m=1}^{n}\frac{1}{m} - \gamma - \ln\left(\frac{\bar{\xi}^2}{2}\right)\right]\right\} \tag{41}$$

for large $\sqrt{B}x$. This result looks a bit more complicated than (36). In the limit $\zeta \to 0$ we recover $G_\mu^{(1)}(xyz, xyz)$, as calculated in [6].

Before we discuss the expressions (36) and (41) in more detail, we will first show that the same results can be derived using a different approach.

5. Parabolic cylinder functions

Results equivalent to those of the previous section can be derived by using the explicit representation of the temperature Green function in terms of parabolic cylinder functions, which are the eigenfunctions of the transverse part of the system Hamiltonian. Because of translation invariance in the y-direction, it is convenient to use a Fourier transform and to write the transverse part of the Hamiltonian as

$$H_\perp(k) = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{1}{2}(Bx - k)^2. \tag{42}$$

For fixed k, this Hamiltonian with boundary condition $\psi(k, x = 0) = 0$ defines an eigenvalue problem. In terms of the eigenfunctions $\psi_n(k, x)$ and the corresponding eigenvalues $E_n(k)$ we define the Fourier-transformed transverse energy Green function [5]

$$G_{\perp,E}(k, x, x') = \sum_n \psi_n(k, x)\psi_n^*(k, x')\,\delta\,[E_n(k) - E] \tag{43}$$

where the eigenfunctions are normalized to $\int_0^\infty \mathrm{d}x|\psi_n(k, x)|^2 = 1$. In terms of this energy Green function the (total) temperature Green function (1) is

$$G_\beta(\boldsymbol{r}, \boldsymbol{r}') = (2\pi\beta)^{-1/2}\exp[-(z - z')^2/2\beta] \times \int_0^\infty \mathrm{d}E\,\mathrm{e}^{-\beta E}\frac{1}{2\pi}\int_{-\infty}^\infty \mathrm{d}k\,\mathrm{e}^{\mathrm{i}k(y-y')}\,G_{\perp,E}(k, x, x'). \tag{44}$$

Performing the inverse Laplace transform as in (4) and using (12), we obtain

$$G_\mu(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{\pi^2}\int_0^\mu \mathrm{d}E\,\frac{\sin\left[\sqrt{2(\mu - E)}(z - z')\right]}{z - z'}\int_{-\infty}^\infty \mathrm{d}k\,\mathrm{e}^{\mathrm{i}k(y-y')}\,G_{\perp,E}(k, x, x'). \tag{45}$$

We now choose $x = x'$, as before, and switch again to the dimensionless variables $\bar{\xi}$, ζ (and a dimensionless Fourier variable $\kappa = k/\sqrt{B}$ as well). In terms of these, the energy Green function is [5]

$$G_{\perp,E}(k, x, x) = \sqrt{B}\sum_n \frac{D^2_{\epsilon_n(k)-1/2}\left[\sqrt{2}(\bar{\xi} - \kappa)\right]}{\int_0^\infty \mathrm{d}\bar{\xi}'\,D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}' - \kappa)\right]}\,\delta[E - B\epsilon_n(\kappa)]. \tag{46}$$

Here $D_\lambda(u)$ is a parabolic cylinder function [15]. Furthermore, $\epsilon_n(\kappa)$ is determined by the boundary condition

$$D_{\epsilon_n(\kappa)-1/2}\left(-\sqrt{2}\kappa\right) = 0. \tag{47}$$

If we now carry out the integration over E in (45), we get

$$G_\mu(xyz, xy'z') = \frac{B^{3/2}}{\pi^2}{\sum_n}' \int_{\kappa_n(\nu)}^\infty \mathrm{d}\kappa\,\mathrm{e}^{\mathrm{i}\kappa\eta}\frac{\sin\left(\sqrt{2[\nu - \epsilon_n(\kappa)]}\,\zeta\right)}{\zeta} \times \frac{D^2_{\epsilon_n(k)-1/2}\left[\sqrt{2}(\bar{\xi} - \kappa)\right]}{\int_0^\infty \mathrm{d}\bar{\xi}'\,D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}' - \kappa)\right]} \tag{48}$$

where $\kappa_n(\nu)$ is determined by $\epsilon_n[\kappa_n(\nu)] = \nu$.

In figure 2 we have plotted the resulting correlation function, which we calculated numerically for several values of $\bar{\xi}$ and for a value of the chemical potential, which corresponds to a completely filled lowest Landau level. Even at moderately small distances from the wall the influence of the confinement on the correlation function is not very big. In the y-direction the correlations become a little stronger. The same is true for the correlations in the z-direction, at least for small ζ. However, the oscillating tail in the correlation function is suppressed by the presence of the wall.

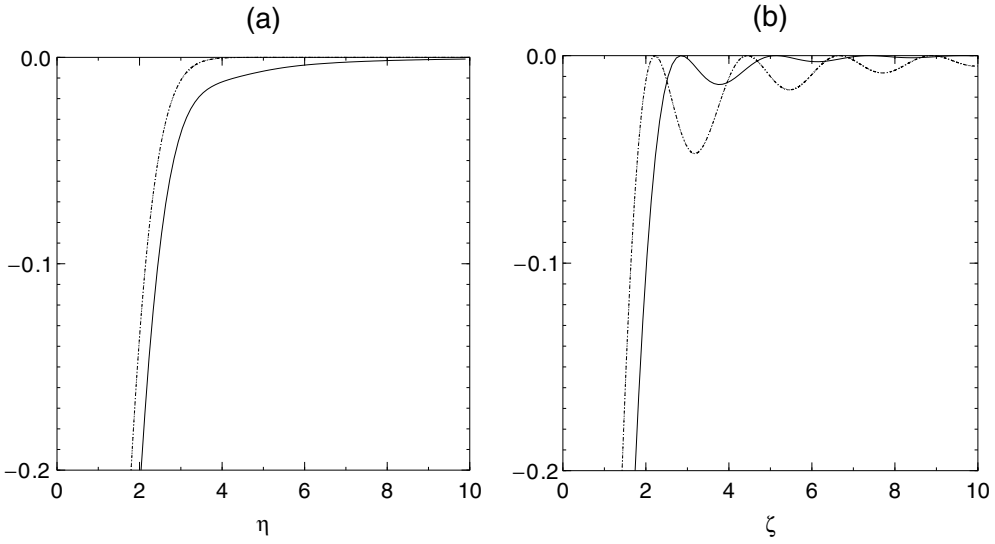

Figure 2. Correlation functions (*a*) $g(xyz, xy'z)$ and (*b*) $g(xyz, xyz')$ for $\bar{\xi} = 1$ (——), $\bar{\xi} = 5$ (— · —) and in the bulk ($\bar{\xi} \to \infty$, - - - -) for $\nu = 1.5$. The curves for $\xi = 5$ and for the bulk are (almost) indistinguishable.

5.1. Case $y \neq y'$

Taking the limit $z' \to z$ or $\zeta \to 0$ in (48) is trivial. The resulting formula contains a phase-factor $\mathrm{e}^{\mathrm{i}\kappa\eta}$, which is absent in the case $y = y'$ as considered in [5]. We first split off the bulk contribution

$$G^b_\mu(xyz, xy'z) = \frac{\sqrt{2}B^{3/2}}{\pi^2} {\sum_n}' \int_{-\infty}^{\infty} \mathrm{d}\kappa\, \mathrm{e}^{\mathrm{i}(\kappa+\bar{\xi})\eta} \frac{\sqrt{\nu - (n+1/2)}}{\sqrt{\pi}\, n!} D_n^2\left(-\sqrt{2}\kappa\right). \tag{49}$$

The remainder $G^c_\mu(xyz, xy'z) = G_\mu(xyz, xy'z) - G^b_\mu(xyz, xy'z)$ gives the correction due to the wall.

In order to obtain an asymptotic expansion of $G^c_\mu(xyz, xy'z)$ for large x we split the integration at $\kappa' = \alpha'\bar{\xi}$, with $0 < \alpha' < 1 - \frac{1}{2}\sqrt{2}$, and $\kappa'' = \alpha''\bar{\xi}$, with $\alpha'' > \frac{1}{2}\sqrt{2}$. As in the case with $\boldsymbol{r} = \boldsymbol{r}'$, the contributions from the intervals $[\kappa_n(\nu), \kappa']$ and $[\kappa'', \infty)$ decay faster than $\exp(-\bar{\xi}^2/2)$. With the help of the asymptotic expressions [5]

$$\epsilon_n(\kappa) - \left(n + \tfrac{1}{2}\right) \approx \frac{1}{\sqrt{\pi}\, n!}\, 2^n\, \mathrm{e}^{-\kappa^2}\, \kappa^{2n+1} \tag{50}$$

$$\left\{\int_0^\infty \mathrm{d}\bar{\xi}'\, D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}' - \kappa)\right]\right\}^{-1}$$
$$\approx \frac{1}{\sqrt{\pi}\, n!} - \frac{1}{\pi (n!)^2}\, 2^{n+1}\, \mathrm{e}^{-\kappa^2}\, \kappa^{2n+1} \left[\sum_{m=1}^{n} \frac{1}{m} - \gamma - \ln\left(\sqrt{2}\kappa\right)\right] \tag{51}$$

and

$$D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi} - \kappa)\right] \approx 2^n\, \mathrm{e}^{-(\bar{\xi}-\kappa)^2}\, (\bar{\xi} - \kappa)^{2n}$$
$$+ \frac{1}{\sqrt{\pi}\, n!}\, 2^{2n+1}\, \mathrm{e}^{-(\bar{\xi}-\kappa)^2}\, \mathrm{e}^{-\kappa^2}\, (\bar{\xi} - \kappa)^{2n}\, \kappa^{2n+1} \ln\left[\sqrt{2}(\bar{\xi} - \kappa)\right] \tag{52}$$

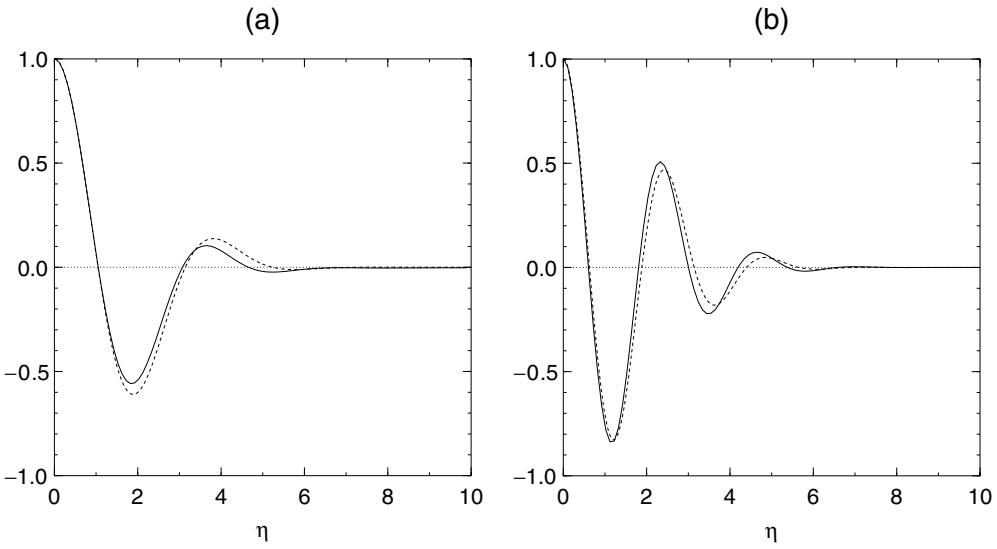

Figure 3. Comparison between numerical data (——) and asymptotic form (- - - -) of the real part of $G^c_\mu(xyz, xy'z)/G^c_\mu(xyz, xyz)$ at (*a*) $\bar{\xi} = 3$ and (*b*) $\bar{\xi} = 5$ for $\nu = 1.5$.

one finds

$$G^c_\mu(xyz, xy'z) \approx \frac{\sqrt{2}B^{3/2}}{\pi^2} \sum_n{}' \frac{2^{2n+1}}{\pi(n!)^2} \sqrt{\nu - (n+1/2)} \times \int_{\kappa'}^{\kappa''} \mathrm{d}\kappa \, \mathrm{e}^{\mathrm{i}\kappa\eta} \, \mathrm{e}^{-\kappa^2} \, \mathrm{e}^{-(\bar{\xi}-\kappa)^2} \, \kappa^{2n+1} (\bar{\xi} - \kappa)^{2n} \, P_n(\kappa, \bar{\xi} - \kappa) \tag{53}$$

with

$$P_n(\kappa, \bar{\xi} - \kappa) = -\left\{ \frac{1}{4[\mu - (n+1/2)]} + \sum_{m=1}^{n} \frac{1}{m} - \gamma - \ln[2\kappa(\bar{\xi} - \kappa)] \right\}. \tag{54}$$

We now expand the integrand in (53) around $\kappa = \bar{\xi}/2$:

$$\int_{\kappa'}^{\kappa''} \mathrm{d}\kappa \, \mathrm{e}^{\mathrm{i}\kappa\eta} \, \mathrm{e}^{-\kappa^2} \, \mathrm{e}^{-(\bar{\xi}-\kappa)^2} \, \kappa^{2n+1} (\bar{\xi} - \kappa)^{2n} \, P_n(\kappa, \bar{\xi} - \kappa) \approx \mathrm{e}^{\mathrm{i}\bar{\xi}\eta/2} \, \mathrm{e}^{-\bar{\xi}^2/2} \, (\bar{\xi}/2)^{4n+1} \, P_n(\bar{\xi}/2, \bar{\xi}/2) \int_{-\infty}^{\infty} \mathrm{d}t \, \mathrm{e}^{\mathrm{i}t\eta - 2t^2} \tag{55}$$

since the main contribution comes from the region around $\kappa = \bar{\xi}/2$, at least for $\bar{\xi} \gg 1$. Evaluating the remaining integral gives us exactly (36).

Note that in this case, instead of $G^{(1)}_\mu(xyz, xy'z)$, we have in fact calculated $G^c_\mu(xyz, xy'z)$. However, since the terms beyond $n = 1$ in the path-decomposition expansion (25) are of higher order, the asymptotic form of $G^c_\mu(xyz, xy'z)$ is in leading order identical to that of $G^{(1)}_\mu(xyz, xy'z)$. In the present case we can easily take along more terms in (54) (see [5]). The expansion around $\kappa = \bar{\xi}/2$ given here is only valid for $\eta \ll \bar{\xi}$. If η is of the same order of magnitude as $\bar{\xi}$, the terms that we left out would not be small compared to the leading term and we would not get the right asymptotics. In figure 3 we compare the results of a numerical evaluation of the exact expression for the real part of

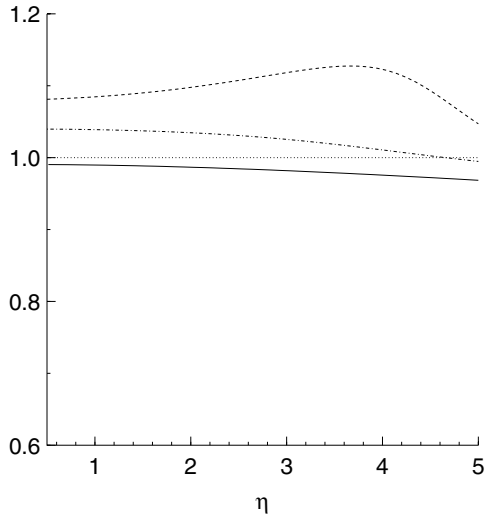

Figure 4. Check of the Gaussian decay, by evaluation of $\sqrt{8}\eta^{-1}[\log(|G^c_\mu(xyz, xy'z)|/|G^c_\mu(xyz, xyz)|)]^{1/2}$, for $\nu = 1.5$ and $\bar{\xi} = 5$ (——), $\bar{\xi} = 4$ (— · —) and $\bar{\xi} = 3$ (- - - -). The dotted line (· · · · · ·) is the asymptotic value according to (36).

the wall correction $G^c_\mu(xyz, xy'z)/G^c_\mu(xyz, xyz)$, which follows from (48) and (49), and the asymptotic form in leading order, as derived in (36). As expected, the agreement is best for small η. The imaginary part of $G^c_\mu(xyz, xy'z)/G^c_\mu(xyz, xyz)$ behaves in a similar way.

A striking feature of the asymptotic expansion (36) is its Gaussian decay proportional to $e^{-\eta^2/8}$. It is slower than the decay of the bulk μ-dependent Green function (13), which is proportional to $e^{-\eta^2/4}$. This somewhat slower decay is indeed corroborated by the numerical evaluation of $G^c_\mu(xyz, xy'z)/G^c_\mu(xyz, xyz)$, as is shown in figure 4. For small η and large $\bar{\xi}$ (i.e. for the regime where (36) holds) the curves converge to the asymptotic value 1.

5.2. Case $z \neq z'$

On setting $y = y'$ in (48) the phase-factor $e^{ik\eta}$ drops out. With the help of the same asymptotic expressions as in the case with $y \neq y'$, and the same splitting of the integration interval, we arrive at the asymptotic expression

$$G^c_\mu(xyz, xy'z) \approx \frac{\sqrt{2}B^{3/2}}{\pi^2} \sum_n{}' \frac{2^{2n+1}}{\pi(n!)^2} \sqrt{\nu - (n+1/2)} \times \int_{\kappa'}^{\kappa''} d\kappa\, e^{-\kappa^2}\, e^{-(\bar{\xi}-\kappa)^2}\, \kappa^{2n+1}\, (\bar{\xi}-\kappa)^{2n}\, Q_n(\kappa, \bar{\xi}-\kappa, \zeta) \tag{56}$$

with

$$Q_n(\kappa, \bar{\xi}-\kappa, \zeta) = -\frac{\cos\left(\sqrt{2[\nu-(n+1/2)]}\,\zeta\right)}{4[\nu-(n+1/2)]} - \frac{\sin\left(\sqrt{2[\nu-(n+1/2)]}\,\zeta\right)}{\sqrt{2[\nu-(n+1/2)]}\zeta} \times \left\{ \sum_{m=1}^{n} \frac{1}{m} - \gamma - \ln[2\kappa(\bar{\xi}-\kappa)] \right\}. \tag{57}$$

Again, the main contribution to the integral comes from the region around $\kappa = \bar{\xi}/2$. Expansion around this point allows us to evaluate the integral, and we recover (41).

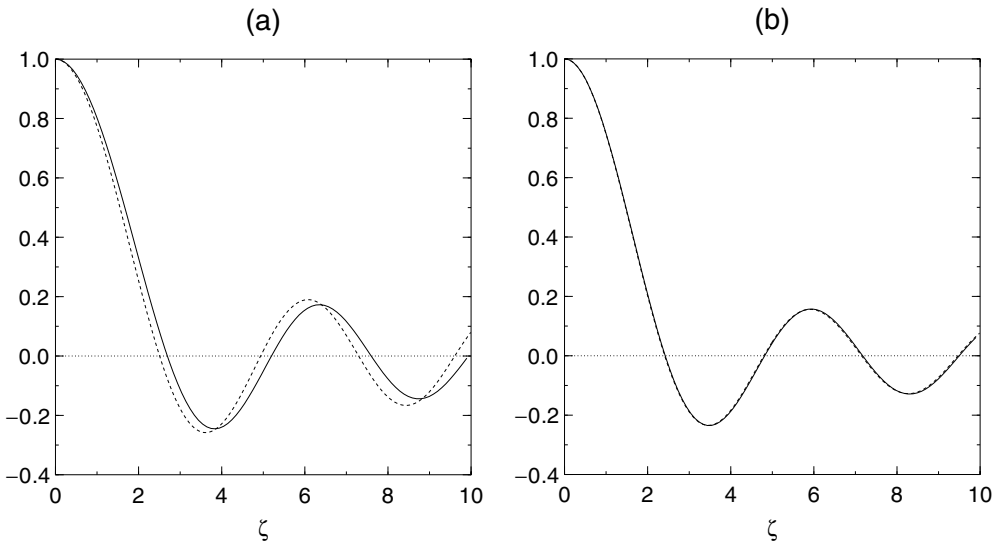

Figure 5. Comparison between numerical data (——) and asymptotic form (- - - -) of $G^c_\mu(xyz, xyz')/G^c_\mu(xyz, xyz)$ at (*a*) $\bar{\xi} = 3$ and (*b*) $\bar{\xi} = 5$ for $\nu = 1.5$. In the latter case the two curves are (almost) indistinguishable.

In figure 5 we compare the asymptotic expression (41) for $G^c_\mu(xyz, xyz')$ with numerical data. As expected, the differences get smaller for increasing values of $\bar{\xi}$.

6. Correlations for large separations $|r - r'|$

The path-decomposition expansion, which we employed in section 4, is expedient for large x only. However, the range of validity of the representation of the previous section is not limited to that regime, so that we can use it for small x as well. In particular, it is helpful in determining the asymptotic behaviour of the correlation function for finite x and large distances between the points of observation, both in the y- and the z-direction.

6.1. Large $|y - y'|$

Let us go back to (48). In order to bring the square-root singularity at $\kappa = \kappa_n(\nu)$ to the fore, we write

$$G_\mu(xyz, xy'z) = \frac{\sqrt{2}B^{3/2}}{\pi^2} {\sum_n}' \int_{\kappa_n(\nu)}^{\infty} \mathrm{d}\kappa\, \mathrm{e}^{\mathrm{i}\kappa\eta} \sqrt{\kappa - \kappa_n(\nu)}\, \phi_n(\kappa) \tag{58}$$

with

$$\phi_n(\kappa) = \frac{\sqrt{\nu - \epsilon_n(\kappa)}}{\sqrt{\kappa - \kappa_n(\nu)}} \frac{D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi} - \kappa)\right]}{\int_0^\infty \mathrm{d}\bar{\xi}'\, D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}' - \kappa)\right]}. \tag{59}$$

The function $\phi_n(\kappa)$ is analytic on the integration interval, and proportional to $\kappa^{2n-1/2}\mathrm{e}^{-\kappa^2}$ for $\kappa \to \infty$. Hence, the asymptotics of (58) for large $|\eta|$ are determined by the lower boundary

of the integral only. With the help of the method of stationary phase [20] we find

$$\int_{\kappa_n(\nu)}^{\infty} \mathrm{d}\kappa\, \mathrm{e}^{\mathrm{i}\kappa\eta}\sqrt{\kappa-\kappa_n(\nu)}\,\phi_n(\kappa) \approx |\eta|^{-3/2}\,\mathrm{e}^{\mathrm{i}[\kappa_n(\nu)\eta+(3\pi/4)\,\mathrm{sgn}(\eta)]}\,\Gamma\left(\frac{3}{2}\right) \times\sqrt{-\frac{\mathrm{d}\epsilon_n(\kappa)}{\mathrm{d}\kappa}}\bigg|_{\kappa=\kappa_n(\nu)} \frac{D^2_{\nu-1/2}\left[\sqrt{2}(\bar{\xi}-\kappa_n(\nu))\right]}{\int_0^{\infty}\mathrm{d}\bar{\xi}'\,D^2_{\nu-1/2}\left[\sqrt{2}(\bar{\xi}'-\kappa_n(\nu))\right]} \tag{60}$$

where the derivative of $\epsilon_n(\kappa)$ is negative. In [5] we have derived the identity

$$\left\{\int_0^{\infty}\mathrm{d}\bar{\xi}'\,D^2_{\epsilon_n(\kappa)-1/2}\left(\sqrt{2}(\bar{\xi}'-\kappa)\right)\right\}^{-1} = -\frac{1}{2\pi}\Gamma^2\left[-\epsilon_n(\kappa)+\frac{1}{2}\right]D^2_{\epsilon_n(\kappa)-1/2}\left(\sqrt{2}\kappa\right)\frac{\mathrm{d}\epsilon_n(\kappa)}{\mathrm{d}\kappa}. \tag{61}$$

This equality allows us to get rid of the normalization integral in (60). In this way we arrive at the following asymptotic expression for the μ-dependent Green function at large $|\eta|$ and finite $\bar{\xi}$:

$$G_\mu(xyz,xy'z) \approx \frac{B^{3/2}\,\mathrm{e}^{\mathrm{i}(3\pi/4)\,\mathrm{sgn}(\eta)}}{2^{3/2}\,\pi^{5/2}}\,\Gamma^2\left(-\nu+\frac{1}{2}\right)\frac{1}{|\eta|^{3/2}}{\sum_n}'\,\mathrm{e}^{\mathrm{i}\kappa_n(\nu)\eta} \times D^2_{\nu-1/2}\left[\sqrt{2}\kappa_n(\nu)\right]D^2_{\nu-1/2}\left[\sqrt{2}(\bar{\xi}-\kappa_n(\nu))\right]\left[-\frac{\mathrm{d}\kappa_n(\nu)}{\mathrm{d}\nu}\right]^{-3/2}. \tag{62}$$

The asymptotic expression simplifies considerably for the special case of a completely filled lowest Landau level, that is for $\nu = 3/2$ and $n = 0$. By using the representation of the parabolic cylinder function in terms of confluent hypergeometric functions [15] we find $\kappa_0(\nu) = 0$ and $\mathrm{d}\kappa_0(\nu)/\mathrm{d}\nu = -\sqrt{\pi}/2$. Employing (61) and inserting $D_1(\sqrt{2}u) = \sqrt{2}u\exp(-u^2/2)$, we arrive at the simple asymptotic expression

$$G_\mu(xyz,xy'z) \approx \frac{4B^{3/2}\,\mathrm{e}^{\mathrm{i}(3\pi/4)\,\mathrm{sgn}(\eta)}}{\pi^{9/4}}\bar{\xi}^2\,\mathrm{e}^{-\bar{\xi}^2}\frac{1}{|\eta|^{3/2}} \tag{63}$$

for large $|\eta|$ and $\nu = 3/2$. The asymptotic behaviour proportional to $|\eta|^{-3/2}$ in (62) and (63) is clearly induced by the presence of the wall, since the decay of the bulk μ-dependent Green function in the y-direction is Gaussian, at least in the presence of a magnetic field (see (13)). The algebraic decay is corroborated by a numerical evaluation of (48). Both results are compared in figure 6.

6.2. Large $|z-z'|$

To determine the asymptotics for large separations in the z-direction we set $y = y'$ in (48). Subsequently, we need to determine the asymptotic behaviour of the imaginary part of the integral

$$I = \int_{\kappa_n(\nu)}^{\infty}\mathrm{d}\kappa\,\mathrm{e}^{\mathrm{i}\sqrt{2[\nu-\epsilon_n(\kappa)]}\,|\zeta|}\,\frac{D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}-\kappa)\right]}{\int_0^{\infty}\mathrm{d}\bar{\xi}'\,D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}'-\kappa)\right]} \tag{64}$$

for large $|\zeta|$. In order to obtain the asymptotics, we split the integration interval at $\kappa' \gg 1$. For $\kappa \geqslant \kappa'$ we can use (50) and (51). In the numerator we insert the asymptotic expression for the parabolic cylinder function

$$D_{\epsilon_n(\kappa)-1/2}\left(\sqrt{2}(\bar{\xi}-\kappa)\right) \approx 2^{n/2+1}(-1)^n\kappa^n\mathrm{e}^{-\kappa^2/2}\,\sinh(\kappa\bar{\xi}-\bar{\xi}^2/2) \tag{65}$$

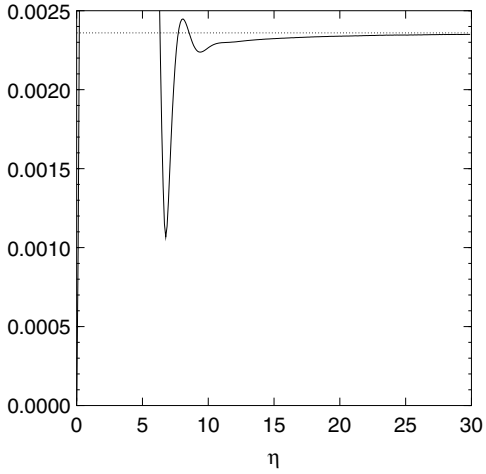

Figure 6. Comparison of $\eta^{3/2}\pi^2|G^c_\mu(xyz, xy'z)|/(\sqrt{2}B^{3/2})$ (——) at $\bar{\xi} = 3$, $\nu = 1.5$ and its asymptotic value (· · · · · ·) for large $|\eta|$, according to (63).

which is valid for large κ and finite $\bar{\xi}$. This relation can be derived by using (50), and the asymptotic relations

$$D_n\left(\sqrt{2}u\right) \approx (-1)^n\, 2^{n/2}\, |u|^n\, \mathrm{e}^{-u^2/2} \tag{66}$$

and

$$\left.\frac{\partial D_\nu\left(\sqrt{2}u\right)}{\partial \nu}\right|_{\nu=n} \approx (-1)^{n+1} 2^{-n/2}\sqrt{\pi}\, n!\, |u|^{-n-1}\, \mathrm{e}^{u^2/2} \tag{67}$$

which are both valid for large negative u. In this way the contribution from $\kappa \geqslant \kappa'$ to (64) becomes

$$I_1 \approx \int_{\kappa'}^{\infty} \mathrm{d}\kappa \exp\left(\mathrm{i}\sqrt{2[\nu-(n+1/2)]}|\zeta| - \mathrm{i}\frac{2^{n-1/2}\,\mathrm{e}^{-\kappa^2}\,\kappa^{2n+1}}{\sqrt{\pi}\, n!\sqrt{\nu-(n+1/2)}}|\zeta|\right) \times \frac{2^{n+2}}{\sqrt{\pi}\, n!}\,\kappa^{2n}\,\mathrm{e}^{-\kappa^2}\sinh^2(\kappa\bar{\xi} - \bar{\xi}^2/2). \tag{68}$$

We now introduce a new integration variable $t = \mathrm{e}^{-\kappa^2}\,\kappa^{2n+1}$, which implies $\kappa \approx \sqrt{-\ln t}$ for large κ. The integral then gets the form

$$I_1 \approx \frac{2^{n+1}}{\sqrt{\pi}\, n!}\,\mathrm{e}^{\mathrm{i}\sqrt{2[\nu-(n+1/2)]}|\zeta|} \int_0^{t'} \mathrm{d}t \exp\left(-\mathrm{i}\frac{2^{n-1/2}|\zeta|}{\sqrt{\pi}\, n!\sqrt{\nu-(n+1/2)}}\, t\right) \times \frac{\sinh^2\left(\sqrt{-\ln t}\,\bar{\xi} - \bar{\xi}^2/2\right)}{-\ln t} \tag{69}$$

with $t' = \mathrm{e}^{-\kappa'^2}\,\kappa'^{2n+1} \ll 1$. Note that there is a logarithmic singularity at $t = 0$.

The asymptotic expansion of an integral of the form

$$\int_0^{t'} \mathrm{d}t\, \mathrm{e}^{\mathrm{i}ut}(-\ln t)^\mu \tag{70}$$

for $t' < 1$ and large u has a contribution from the lower boundary and the upper boundary. The contribution from the lower boundary is [21]

$$\frac{\mathrm{i}}{u}\sum_{r=0}^{\infty}(-1)^r\binom{\mu}{r}\left[\sum_{k=0}^{r}\binom{r}{k}\Gamma^{(k)}(1)\left(\frac{\pi\mathrm{i}}{2}\right)^{r-k}\right](\ln u)^{\mu-r} \tag{71}$$

where $\Gamma^{(k)}(u)$ is the kth derivative of $\Gamma(u)$. For functions $f(-\ln t)$ that can be expressed as a Laurent series in $\sqrt{-\ln t}$ for small t, one obtains the asymptotics of the integral

$$\int_0^{t'}\mathrm{d}t\,\mathrm{e}^{\mathrm{i}ut}\,f(-\ln t) \tag{72}$$

for large u by integrating the series term-by-term by means of (71). In fact, the contribution from the lower boundary of the integral is

$$\frac{\mathrm{i}}{u}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}\left[\sum_{k=0}^{r}\binom{r}{k}\Gamma^{(k)}(1)\left(\frac{\pi\mathrm{i}}{2}\right)^{r-k}\right]\frac{\mathrm{d}^r f(\ln u)}{\mathrm{d}(\ln u)^r}. \tag{73}$$

Let us now return to (69), which contains an integral that has the form of the complex conjugate of (72). For this particular case one has $f(-\ln t) = \sinh^2(\sqrt{-\ln t}\bar{\xi} - \bar{\xi}^2/2)/(-\ln t)$ and $u = 2^{n-1/2}|\zeta|/\left[\sqrt{\pi}\,n!\sqrt{\nu-(n+1/2)}\right]$. The term with $r = 0$ in (73) dominates the contribution from the lower boundary to the asymptotics, since $f(\ln u)$ is much bigger than all its derivatives when u is large. Using this fact, we get for the contribution from the lower boundary ($t = 0$) to the asymptotics of (69)

$$\begin{aligned}-\mathrm{i}\frac{2^{n+1}}{\sqrt{\pi}\,n!\,u}\,\mathrm{e}^{\mathrm{i}\sqrt{2[\nu-(n+1/2)]}|\zeta|}\frac{\sinh^2\left(\sqrt{\ln u}\bar{\xi}-\bar{\xi}^2/2\right)}{\ln u}\\ \approx -\mathrm{i}\frac{2\sqrt{2[\nu-(n+1/2)]}}{|\zeta|\ln|\zeta|}\,\mathrm{e}^{\mathrm{i}\sqrt{2[\nu-(n+1/2)]}|\zeta|}\sinh^2\left(\sqrt{\ln|\zeta|}\bar{\xi}-\bar{\xi}^2/2\right).\end{aligned} \tag{74}$$

The contribution from the upper boundary ($t = t'$) to the asymptotics of (69) can be found by use of the same theorem from [21]. It is expected to cancel against the contribution from the end-point $\kappa = \kappa'$ of the integral

$$I_2 = \int_{\kappa_n(\nu)}^{\kappa'}\mathrm{d}\kappa\,\mathrm{e}^{\mathrm{i}\sqrt{2[\nu-\epsilon_n(\kappa)]}|\zeta|}\frac{D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}-\kappa)\right]}{\int_0^{\infty}\mathrm{d}\bar{\xi}'\,D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi}'-\kappa)\right]}. \tag{75}$$

Indeed, it is not too difficult to show that it does, by evaluating the contribution from the end-point at $\kappa = \kappa'$ in I_2 with the help of standard techniques [20]. Finally, we have to check whether the end-point at $\kappa = \kappa_n(\nu)$ contributes to the asymptotics of I_2. The phase has a square-root singularity at that point. Using this fact, one finds that all terms in the asymptotic expansion that originate from the lower boundary of I_2 are real. Hence, they drop out when taking the imaginary part of I.

Collecting our results, we have derived the asymptotic equality

$$\mathrm{Im}\,I \approx -\frac{2\sqrt{2[\nu-(n+1/2)]}}{|\zeta|\ln|\zeta|}\cos\left(\sqrt{2[\nu-(n+1/2)]}\zeta\right)\sinh^2\left(\sqrt{\ln|\zeta|}\bar{\xi}-\bar{\xi}^2/2\right) \tag{76}$$

which is valid for large $|\zeta|$. As a consequence we obtain the μ-dependent Green function

$$\begin{aligned}G_\mu(xyz,xyz') \approx -\frac{2B^{3/2}}{\pi^2}\frac{\sinh^2\left(\sqrt{\ln|\zeta|}\bar{\xi}-\bar{\xi}^2/2\right)}{\zeta^2\ln|\zeta|}\\ \times{\sum_n}'\sqrt{2[\nu-(n+1/2)]}\cos\left(\sqrt{2[\nu-(n+1/2)]}\zeta\right)\end{aligned} \tag{77}$$

for large $|\zeta|$. This asymptotic form is valid at large separations $|z - z'|$ and a finite distance x from the wall. A comparison with (13) shows that the asymptotic behaviour changes substantially owing to the presence of the wall. The simple algebraic tail (modulated by a goniometric factor) which is valid in the bulk, is replaced by a more subtle decay involving a logarithmic dependence on $|z - z'|$.

7. Correlations near the wall

Let us return again to (48). Using (61) we may write it as

$$G_\mu(xyz, xy'z') = -\frac{B^{3/2}}{2\pi^3}{\sum_n}' \int_{\kappa_n(\nu)}^{\infty} \mathrm{d}\kappa\, \mathrm{e}^{\mathrm{i}\kappa\eta} \frac{\sin\left(\sqrt{2[\nu - \epsilon_n(\kappa)]}\,\zeta\right)}{\zeta} \Gamma^2\left[-\epsilon_n(\kappa) + \frac{1}{2}\right]$$
$$\times D^2_{\epsilon_n(\kappa)-1/2}\left(\sqrt{2}\kappa\right) D^2_{\epsilon_n(\kappa)-1/2}\left[\sqrt{2}(\bar{\xi} - \kappa)\right] \frac{\mathrm{d}\epsilon_n(\kappa)}{\mathrm{d}\kappa}. \tag{78}$$

For small $\bar{\xi}$ the last parabolic cylinder function can be expanded in a Taylor series. According to (47) the zeroth-order term of this series vanishes, so that the leading term for small $\bar{\xi}$ is of first order in $\bar{\xi}$. The derivative of the parabolic cylinder function occurring in this term fulfils the Wronskian identity [15]

$$D_{\epsilon_n(\kappa)-1/2}\left(\sqrt{2}\kappa\right) \frac{\partial}{\partial\kappa} D_\nu\left(-\sqrt{2}\kappa\right)\Big|_{\nu=\epsilon_n(\kappa)-1/2} = \frac{2\sqrt{\pi}}{\Gamma\left[-\epsilon_n(\kappa) + \frac{1}{2}\right]} \tag{79}$$

where (47) has been used again. Solving for the derivative we obtain

$$G_\mu(xyz, xy'z') \approx -\frac{2B^{3/2}\bar{\xi}^2}{\pi^2}{\sum_n}' \int_{\kappa_n(\nu)}^{\infty} \mathrm{d}\kappa\, \mathrm{e}^{\mathrm{i}\kappa\eta} \frac{\sin\left(\sqrt{2[\nu - \epsilon_n(\kappa)]}\zeta\right)}{\zeta} \frac{\mathrm{d}\epsilon_n(\kappa)}{\mathrm{d}\kappa}. \tag{80}$$

The right-hand side can be simplified further by introducing the integration variable ϵ instead of κ. In this way we arrive at the following approximate form of the μ-dependent Green function near the wall:

$$G_\mu(xyz, xy'z') \approx \frac{2B^{3/2}\bar{\xi}^2}{\pi^2}{\sum_n}' \int_{n+1/2}^{\nu} \mathrm{d}\epsilon\, \mathrm{e}^{\mathrm{i}\kappa_n(\epsilon)\eta} \frac{\sin\left[\sqrt{2(\nu - \epsilon)}\zeta\right]}{\zeta}. \tag{81}$$

As before we consider the cases $\eta \neq 0$ and $\zeta \neq 0$ separately.

7.1. Case $y \neq y'$

For $z' \to z$ the Green function becomes

$$G_\mu(xyz, xy'z) \approx \frac{2^{3/2}B^{3/2}\bar{\xi}^2}{\pi^2}{\sum_n}' \int_{n+1/2}^{\nu} \mathrm{d}\epsilon\, \mathrm{e}^{\mathrm{i}\kappa_n(\epsilon)\eta}\sqrt{\nu - \epsilon}. \tag{82}$$

For large $|\eta|$ the asymptotic behaviour of the integral is determined by the value of the integrand near the upper boundary. One finds

$$G_\mu(xyz, xy'z) \approx \frac{\sqrt{2}B^{3/2}\,\mathrm{e}^{\mathrm{i}(3\pi/4)\,\mathrm{sgn}(\eta)}\bar{\xi}^2}{\pi^{3/2}|\eta|^{3/2}}{\sum_n}' \mathrm{e}^{\mathrm{i}\kappa_n(\nu)\eta}\left[-\frac{\mathrm{d}\kappa_n(\nu)}{\mathrm{d}\nu}\right]^{-3/2}. \tag{83}$$

The same result can be obtained from (62) by expanding the last parabolic cylinder function for small $\bar{\xi}$ and using (79).

The asymptotic decay proportional to $|\eta|^{-3/2}$ in (83) seems to be at variance with the behaviour of the correlations in the field-free case. For the latter case one finds

$$G_\mu(xyz, xy'z) \approx -\frac{4\mu x^2}{\pi^2(y - y')^3}\sin\left[\sqrt{2\mu}(y - y')\right] \tag{84}$$

by expanding (24) with (21) for small x and retaining the terms dominant at large $|y - y'|$. Hence, the correlations near the wall decay faster in the field-free case than in the case with field. It should be noted that (83) is valid for $|\eta| \gg 1$ or $|y - y'| \gg B^{-1/2}$. For B tending to 0 the region of validity thus shifts towards ∞. Furthermore, the number of terms in the sum becomes very large for small B (since ν gets large), whereas the factor in front tends to 0, so that taking the limit $B \to 0$ in (83) is not trivial.

The decay of the field-free correlations can be derived in the present context by starting from (82). Let us interchange the summation and the integral and split the latter at $\epsilon' \gg 1$. We get

$$\sum_n{}' \int_{n+1/2}^{\nu} \mathrm{d}\epsilon\, \mathrm{e}^{\mathrm{i}\kappa_n(\epsilon)\eta}\sqrt{\nu-\epsilon} = \int_{1/2}^{\epsilon'} \mathrm{d}\epsilon\, \sqrt{\nu-\epsilon} \sum_{n=0}^{[\epsilon-1/2]} \mathrm{e}^{\mathrm{i}\kappa_n(\epsilon)\eta} + \int_{\epsilon'}^{\nu} \mathrm{d}\epsilon \sqrt{\nu-\epsilon} \sum_{n=0}^{[\epsilon-1/2]} \mathrm{e}^{\mathrm{i}\kappa_n(\epsilon)\eta}. \tag{85}$$

In the second part at the right-hand side the sum consists of many terms for each ϵ. As discussed in the appendix, the values of $\kappa_n(\epsilon)$ are located in the interval $\left[-\sqrt{2\epsilon}, \sqrt{2\epsilon}\right]$, at least for all $n \leqslant [\epsilon - 1/2] - 1$. As a consequence, their average separation goes to zero proportional to $1/\sqrt{\epsilon}$. It is therefore expedient to replace the summation over n by an integration over a continuous variable $\sigma = \kappa_n(\epsilon)/\sqrt{2\epsilon}$. The second part in (85) then becomes

$$\int_{\epsilon'}^{\nu} \mathrm{d}\epsilon \sqrt{\nu-\epsilon} \int_{-1}^{1} \mathrm{d}\sigma\, \mathrm{e}^{\mathrm{i}\sqrt{2\epsilon}\sigma\eta}\rho(\epsilon,\sigma) \tag{86}$$

with $\rho(\epsilon,\sigma)\,\mathrm{d}\sigma$ the number of values of $\kappa_n(\epsilon)$ in the interval $\left[\sqrt{2\epsilon}\sigma, \sqrt{2\epsilon}(\sigma + \mathrm{d}\sigma)\right]$. For large $|\eta|$ the dominant contribution in the integral over σ comes from the endpoints. Since the density $\rho(\epsilon,\sigma)$ at the endpoints $\sigma = \pm 1$ is $2^{3/2}\epsilon(1 \mp \sigma)^{1/2}/\pi$ (see the appendix), the expression (86) is for large $|\eta|$

$$\frac{2^{3/4}}{\pi^{1/2}|\eta|^{3/2}} \int_{\epsilon'}^{\nu} \mathrm{d}\epsilon\, \sqrt{\nu-\epsilon}\epsilon^{1/4} \cos\left(\sqrt{2\epsilon}|\eta| - 3\pi/4\right). \tag{87}$$

The integral over ϵ can likewise be evaluated for large $|\eta|$, as once again only the endpoints of the integral contribute. The upper boundary gives

$$-\sqrt{2}\frac{\nu}{\eta^3}\sin\left(\sqrt{2\nu}\eta\right) \tag{88}$$

while the contribution from the lower boundary in (87) will not be needed.

As a final step we have to consider the first part in (85). Here the summation can be replaced by an integration for $\epsilon \gg 1$ only. Of course, the contribution of the upper boundary to the asymptotic expression has to cancel that of the lower boundary in (87), since the final result should not depend on the choice of ϵ'. The lower boundary of the integral over ϵ in the first part of (85) does not contribute either. In fact, only the term with $n = 0$ survives for ϵ near $1/2$ and the factor $\kappa_0(\epsilon)$ goes to infinity for $\epsilon \to 1/2$, so that for large $|\eta|$ the contribution from the lower boundary damps out by interference.

Collecting the results, we have found that for $B \to 0$ and large $|\eta|$ the asymptotic behaviour of the μ-dependent Green function near the wall is given by (82) with (88) inserted:

$$G_\mu(xyz, xy'z) \approx -\frac{4B^{3/2}\nu\bar{\xi}^2}{\pi^2\eta^3}\sin\left(\sqrt{2\nu}\eta\right). \tag{89}$$

After restoring the field-independent variables this result coincides with (84).

7.2. Case $z \neq z'$

In this case the dependence on $\kappa_n(\epsilon)$ drops out from (81)

$$G_\mu(xyz, xyz') \approx \frac{2B^{3/2}\bar{\xi}^2}{\pi^2}{\sum_n}' \int_{n+1/2}^{\nu} \mathrm{d}\epsilon \, \frac{\sin\left[\sqrt{2[\nu - \epsilon_n(\kappa)]}\zeta\right]}{\zeta}. \tag{90}$$

The integral is elementary. As a result we get for the Green function at small x

$$G_\mu(xyz, xyz') \approx -\frac{2B^{3/2}\bar{\xi}^2}{\pi^2\zeta^3}{\sum_n}' \left\{\sqrt{2[\nu - (n+1/2)]}\zeta \cos\left[\sqrt{2[\nu - (n+1/2)]}\,\zeta\right] - \sin\left[\sqrt{2[\nu - (n+1/2)]}\,\zeta\right]\right\}. \tag{91}$$

By taking the limit $\zeta \to 0$ we obtain a simple form for the particle density near the wall:

$$\rho_\mu = G_\mu(\boldsymbol{r}, \boldsymbol{r}) \approx \frac{2^{5/2}B^{3/2}\bar{\xi}^2}{3\pi^2}{\sum_n}' [\nu - (n+1/2)]^{3/2} \tag{92}$$

which should be compared to the bulk density (14).

For large separations the first term between the curly brackets in (91) is dominant, so that the μ-dependent Green function for large separations and small distance from the wall becomes

$$G_\mu(xyz, xyz') \approx -\frac{2B^{3/2}\bar{\xi}^2}{\pi^2\zeta^2}{\sum_n}' \sqrt{2[\nu - (n+1/2)]} \cos\left[\sqrt{2[\nu - (n+1/2)]}\zeta\right]. \tag{93}$$

The same expression can also be found from (77), by putting $\bar{\xi} \ll 1$ and expanding the hyperbolic function for small values of its argument.

For small B it is convenient to return to the original variables x, $z - z'$, μ and to introduce the new integration variable $u = \epsilon B$ in (90). Replacing in addition the summation by an integration we get

$$G_\mu(xyz, xyz') \approx \frac{2x^2}{\pi^2} \int_0^\mu \mathrm{d}t \int_t^\mu \mathrm{d}u \, \frac{\sin\left[\sqrt{2(\mu - u)}(z - z')\right]}{z - z'}. \tag{94}$$

Performing the integrations we obtain the expression

$$G_\mu(xyz, xyz') \approx -\frac{2\mu x^2}{\pi^2(z - z')^3}\left\{2\sin\left[\sqrt{2\mu}(z - z')\right] + \frac{6}{\sqrt{2\mu}(z - z')}\cos\left[\sqrt{2\mu}(z - z')\right] - \frac{3}{\mu(z - z')^2}\sin\left[\sqrt{2\mu}(z - z')\right]\right\} \tag{95}$$

for the μ-dependent Green function at positions near the wall in the field-free case. It can easily be checked that the same expression is found by expanding the general form (24) for small x.

At large separations $|z - z'|$ the dominant term in (95) is the first one. It agrees with (84), when y and z are interchanged.

8. Discussion and conclusion

Our results show that the presence of a magnetic field and a wall leads to remarkable changes in the pair correlation function of a completely degenerate non-interacting electron gas. In the bulk the correlation function follows from (7), with the μ-dependent Green functions (13) and (21) for the cases with and without field. Whereas the correlation function is isotropic

and decays algebraically ($\propto r^{-4}$) in the field-free case, it becomes anisotropic with a Gaussian dependence in the transverse direction and an algebraic one ($\propto r^{-2}$) in the parallel direction.

In the neighbourhood of a plane hard wall the pair correlation function becomes anisotropic even for the field-free case. Its form, which follows directly from a reflection principle, is given by (7) with (24). For two positions far apart, but at equal distance from the wall, the ensuing correlation function decays algebraically, as in the bulk. However, the decay is proportional to r^{-6}, as follows from (84).

If in addition to the wall a magnetic field is present, the pair correlation function in the vicinity of the wall becomes anisotropic for two reasons, as both the field and the wall break the symmetry. The general expression for the correlation function, for arbitrary field strengths and arbitrary distances from the wall, follows by substitution of (48) in (7). Relatively far from the wall the corrections to the bulk correlation function are still small. The first-order correction terms were given in (36) and (41), and checked numerically in figures 3 and 5. Near the wall the correlation function—and in particular its tail for large separations—is modified considerably compared to its form in the bulk. In fact, the qualitative difference between the behaviour in the directions parallel and transverse to the field, which is a prominent feature of the bulk correlation function, is lost in the vicinity of the wall. In both directions the tails become algebraic, albeit with a different exponent. This is seen by inspecting (62) (or (83)) for the transverse direction and (77) (or (93)) for the parallel direction. In the former case the decay is proportional to r^{-3}, whereas in the latter it is proportional to r^{-4}. Qualitatively, the change in the decay of the transverse correlation function from Gaussian in the bulk to algebraic near the wall can be understood in a semi-classical picture. In the bulk the cyclotron motion of the particles leads to a strong localization of the correlations, which is associated with a Gaussian decay. On approaching the wall, the so-called 'skipping orbits' along the wall become important. They lead to a delocalization effect in the particle motion, which manifests itself as an increase in the range of the correlations. In this way, the cross-over to an algebraic decay of the correlation function finds an explanation.

In conclusion, we have shown that for a non-interacting electron gas the influence of a wall on the correlations is quite considerable, especially in the presence of a magnetic field. It would be interesting to determine the edge effects in the correlations for an interacting electron gas in a magnetic field.

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Appendix. Zeros of the parabolic cylinder functions for large argument and index

The zeros of $D_\lambda(z)$ satisfy the inequality $|z| < 2\sqrt{\lambda + 1/2}$, with the possible exception of a single negative zero [22]. For $\lambda \gg 1$ the zeros in the neighbourhood of $|z| = 2\sqrt{\lambda + 1/2}$ can be determined by approximating the parabolic cylinder functions by Airy functions.

For positive z and $\sigma = z/\left[2\sqrt{\lambda + 1/2}\right]$ slightly smaller than 1, one has [22]

$$D_\lambda(z) \approx 2^{\lambda/2+1/3}\Gamma\left(\frac{1}{2}\lambda + \frac{1}{2}\right)\left(\lambda + \frac{1}{2}\right)^{1/6}\left(\frac{-\tau}{1-\sigma^2}\right)^{1/4}\mathrm{Ai}\left(\left[4\left(\lambda + \frac{1}{2}\right)\right]^{2/3}\tau\right) \qquad \text{(A.1)}$$

with τ given by

$$\tau = -\left\{\frac{3}{2}\left[\frac{1}{4}\arccos(\sigma) - \frac{1}{4}\sigma\sqrt{1-\sigma^2}\right]\right\}^{2/3} \approx -2^{-1/3}(1-\sigma). \tag{A.2}$$

For $x \gg 1$ the Airy function may be approximated as [22]

$$\mathrm{Ai}(-x) \approx \frac{1}{\sqrt{\pi}x^{1/4}}\sin\left(\frac{2}{3}x^{3/2} + \frac{\pi}{4}\right). \tag{A.3}$$

Using this approximation in (A.1), one finds the zeros of the parabolic cylinder function from the zeros of the sine function. In terms of the variable σ, the zeros near $\sigma = 1$ are found as

$$\sigma_m = 1 - \frac{1}{2}\left[\frac{3\pi(m-1/4)}{2(\lambda+1/2)}\right]^{2/3} \tag{A.4}$$

with positive integer m. Since λ is large, the zeros are closely spaced. In view of the main text we introduce $\varepsilon = \lambda + 1/2$ instead of λ. Writing the number of zeros between σ and $\sigma + \mathrm{d}\sigma$ as $\rho(\varepsilon,\sigma)\,\mathrm{d}\sigma$ we get

$$\rho(\varepsilon,\sigma) = \frac{2^{3/2}}{\pi}\varepsilon(1-\sigma)^{1/2} \tag{A.5}$$

as the density of zeros near $\sigma = 1$.

Similarly, for negative z and σ slightly bigger than -1 the parabolic cylinder function can be written as [22]

$$D_\lambda(z) \approx 2^{\lambda/2+1/3}\Gamma\left(\frac{1}{2}\lambda + \frac{1}{2}\right)\left(\lambda + \frac{1}{2}\right)^{1/6}\left(\frac{-\tau}{1-\sigma^2}\right)^{1/4} \times \left\{\cos(\pi\lambda)\,\mathrm{Ai}\left(\left[4\left(\lambda+\frac{1}{2}\right)\right]^{2/3}\tau\right) - \sin(\pi\lambda)\,\mathrm{Bi}\left(\left[4\left(\lambda+\frac{1}{2}\right)\right]^{2/3}\tau\right)\right\} \tag{A.6}$$

with $\tau \approx -2^{-1/3}(1+\sigma)$. With the use of (A.3) and the analogous relation

$$\mathrm{Bi}(-x) \approx \frac{1}{\sqrt{\pi}x^{1/4}}\cos\left(\frac{2}{3}x^{3/2} + \frac{\pi}{4}\right) \tag{A.7}$$

(valid for large x), one obtains the zeros of $D_\lambda(z)$ near $\sigma = -1$ as

$$\sigma_m = -1 + \frac{1}{2}\left[\frac{3\pi(m+\lambda-1/4)}{2(\lambda+1/2)}\right]^{2/3} \tag{A.8}$$

for integer m with $m \geqslant -\lambda + 1/4$. The density of the zeros is found to be

$$\rho(\varepsilon,\sigma) = \frac{2^{3/2}}{\pi}\varepsilon(1+\sigma)^{1/2} \tag{A.9}$$

for σ near -1.

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